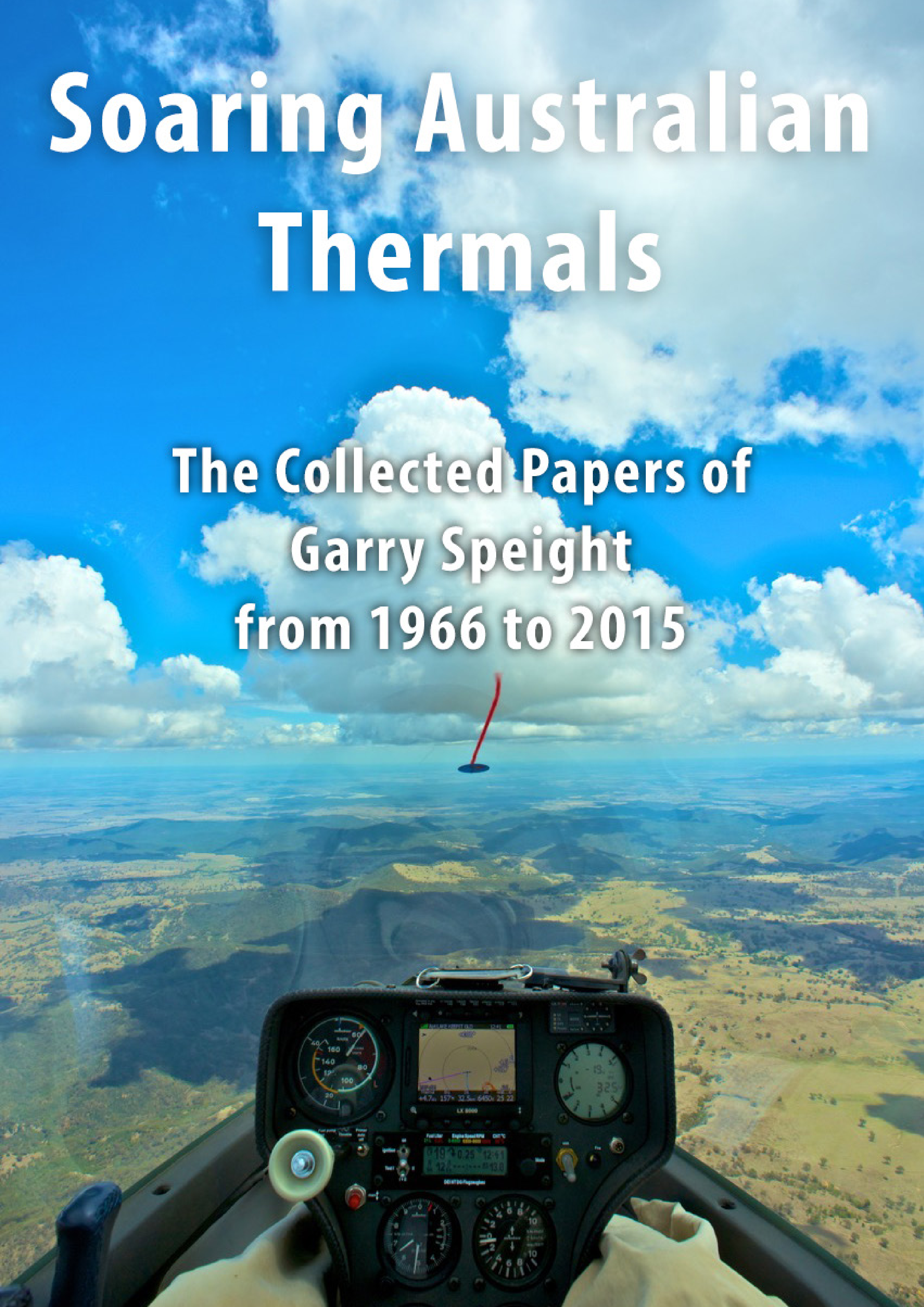
Soaring Australian Thermals

The Collected Papers of Garry Speight from 1966 to 2015



Probability In Cross-Country Flying

Commentaries By Garry Speight

Originally published in Australian Gliding, February 1983

Commentary 1

It was just twenty years ago this month that the following article was published in Sailplane and Gliding magazine. The ideas in it were novel at the time and are still little appreciated, even by competition pilots. Yet the logic is quite unanswerable, and the article puts it beyond doubt that the classical MacCready theory, which takes no account of the probability of failing to find the next thermal, gives only half the story about cross-country strategy.

Since no-one has done similar calculations for modern gliders, the numbers are republished just as in the original article. The Slingsby Swallow was a popular single seater of the day with a claimed best glide ratio of 26:1 at 40 knots, while the Skylark 3, a recent World Championship winner, was rated at 36:1, although Derek Piggott suggests that the actual best glide ratios were about 23:1 and 29:1 respectively. For comparable performance in Australian gliders one can think of a Long-wing Kookaburra and a Schneider Ka6.

Incidentally, the thermal strength, height, and spacing chosen in the example seem about right for spring or autumn soaring conditions at some Australian gliding sites.

A Stochastic Cross-Country Or Festina Lente

by Anthony Edwards

Cambridge University Gliding Club

(Reprinted with permission from *Sailplane and Gliding*, February 1963)

"Whatever do you mean by that?"

"By what?"

"A stochastic cross-country? What does 'stochastic' mean?"

"It means that there is an element of chance in the flight: you might not reach your goal."

"But all flights are like that. "

"Yes. "

"Then why bother to call them by a long word when everyone knows this fact?"

"Well, it's like this ..."

Every cross-country pilot knows that his primary task is to stay up. Only when he is reasonably satisfied about this can he start thinking about the best-speed-to fly, and why Little Rissington hasn't turned up yet, and other such things. And yet, when he comes to work out his best speed, he will certainly not take into account, mathematically, the possibility of a premature landing, although he will do so in his mind ("Better not fly as fast as that . . . might get too low"). But there is no reason why he shouldn't feed the chance, or stochastic element into his calculator. Much is known about Stochastic Processes nowadays, and in this article I want to introduce them to gliding in a very simple example: so simple, in fact, as to be rather unrealistic. But one has to start somewhere.

Today there is no wind. Thin cumulus are randomly dotted over the sky, and I have declared Little Rissington. I am determined not to stray from my track, and a cursory glance at the clouds reveals that thermals will be randomly spaced along the route, every d ft. on average. My operational height-band will be h ft. deep, and — another glance upwards — my rate of climb in thermals will be u f.p.s. And, best of all, no down between thermals! Since Little Rissington is nd ft. away, I'll need about n thermals to get me there. And I mustn't forget my glider — she sinks at $s = Av^3 + B/v$ f.p.s. when flown at v f.p.s. All ready? Right! Hook on, and let's go.

The distance between adjacent thermals is a random variable, x, which is evidently exponentially distributed with probability density $(1/d)exp(-x/d)$. (Help! He's in cloud already!) If you don't know about these things, just shut your eyes for the next few minutes. Now consider the glide from top of one thermal to the next one, x ft. distant, during which the glider is flown

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at v f.p.s. The glide takes x/v seconds, and thus consumes sx/v, or $x(Av^2 + B/v^2)$, feet of height. If this loss exceeds h ft., the glider will land; that is, if x exceeds $h/(Av^2 + B/v^2)$. But the probability of this happening is

$$\frac{1}{d}\int_{\frac{h}{Av^2+B/v^2}}^{\infty} e^{-\frac{x}{d}}\,dx$$

which equals $exp[-h/d(Av^2 + B/v^2)]$. Thus the probability of still being airborne after n glides between thermals (which, you may remember, will take me to Little Rissington) is

$P = (1-exp[-h/d(Av^2 + B/v^2)])^n$.

This is the probability of my reaching the goal. A little thought shows that it has a maximum at $v = (B/A)^{0.25}$, which is the speed for best gliding angle, as The Soaring Pilot will tell you. This is as it should be, and we deduce that the maximum probability of arrival is

$(1-exp(-h/(2d\,(AB)^{0.5}))^n$.

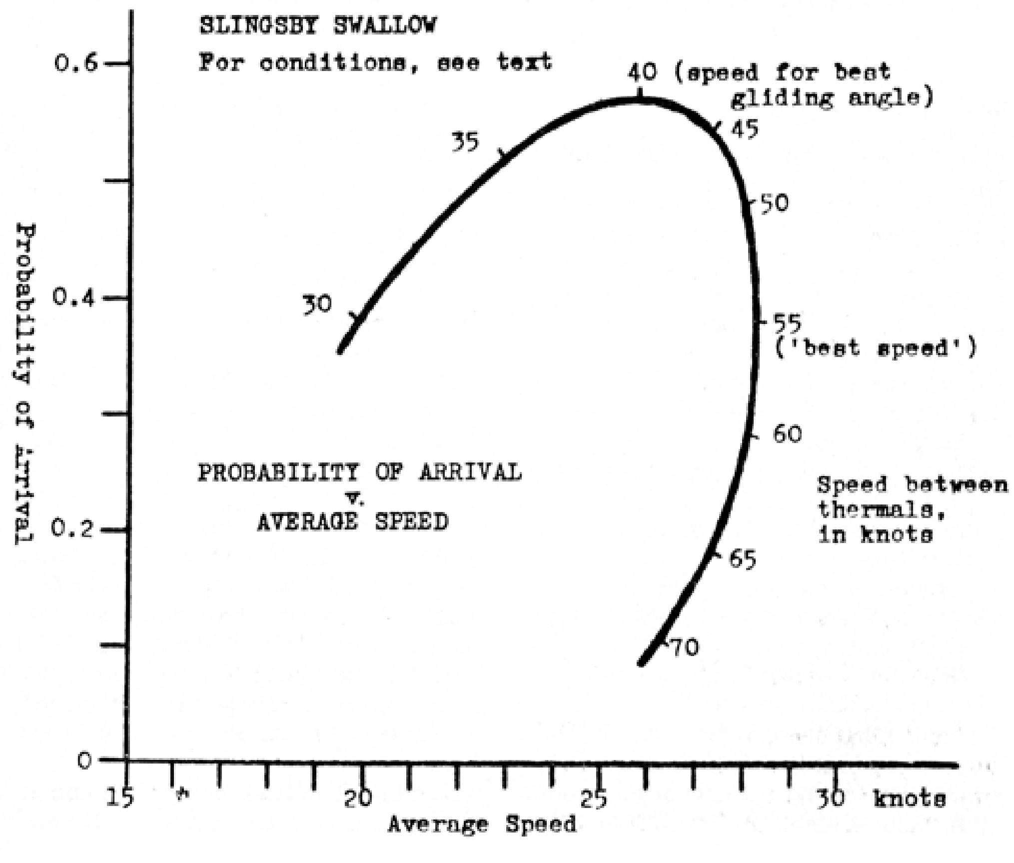
The Soaring Pilot also tells us that the average cross country speed is

$(w = uv/(u + Av^3 + B/v))$.

In order to maximise this I would have to fly faster than my best-gliding-angle speed, as everyone knows, but the probability of my reaching the goal would then be reduced. By how much? Let's look at an actual example.

Suppose Little Rissington is 100 km away, and the thermals are four miles apart on average, d is thus about 21,000 ft and I will need about n =16 thermals. Suppose the operational height-band, h, is 3000 ft, and the rate of climb in thermals, u is 5 f.p.s. If my glider is a Swallow we may guess A=4.5x10^{-6} and B=100 roughly.

Now the last two equations relate the probability of arrival, P, to the average speed, w, by means of the parameter, v. We may therefore draw a graph of P against w, keeping an eye on v at the same time. I have done this in the figure. We



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see that if I fly at the best gliding-angle speed, 40 knots, the probability of arrival is 0.57, but if I fly at the "best-speed-to-fly", 55 knots, the probability is only 0.38, and the average speed has only gone up two-and-a-half knots. An increase of 10% in the average speed costs a reduction of 33% in the probability of arrival. Is it worth it? Well, that depends upon the object of the flight, whether it is a race or not, and, if it is, what marking system is being used. The expectation of points on any given system can be maximised and the appropriate speed-to-fly found.

A more striking deduction from the graph is what happens around the "best- speed-to-fly". It is often said, quite truly, that so long as one "stuffs the nose down" in between thermals, one will come to within a knot or two of the best possible average speed. Thus, in our example, all speeds between 46 and 65 knots lead to cross-country speeds within one knot of the maximum. But look what happens to the probability of arrival: it ranges from 0.54 to 0.18 — a factor of three!

It is interesting to compare a Swallow's maximum probability of arrival with that of a Skylark 3, for which AB must be about 2.5×10^{-4} compared with a Swallow's 4.5×10^{-4}. It turns out to be 0.84, as against 0.57.

From all of which we may draw two conclusions: if you want to get to Little Rissington, go by Skylark; and, whatever your mount, *festina lente*! We are not all free to choose our glider, but we can all choose our own tactics. Stochastic theory clearly has something to contribute to the theory of tactics, and I hope other pipe- dreamers will continue the investigation. One immediate application is to the task of handicapping, which it could change from an art to a science.

Commentary 2 by Garry Speight

Anthony Edwards' hopes for the application of his ideas have come to very little in the twenty years since he expressed them. There has been no development of stochastic theory in gliding textbooks or articles: even Reichmann devotes only one paragraph to "probability" and gives no useful advice on the subject. No-one has collated any of the data that could be taken from barograph traces concerning the frequency distributions of thermal strengths and inter-thermal distances on a given day.

Handicapping calculations still take no account whatever of the high likelihood of out-landing that the pilot of a low- performance glider faces on every inter- thermal glide. When he lands out, it is assumed that his achieved distance should be proportional to his MacCready speed: this merely covers the case of the task being over-set. Competition scoring usually contains some esoteric calculation relating to the proportion of gliders finishing the course, but this appears to be quite arbitrary. As Anthony Edwards points out, if a pilot knows the basis of the scoring system, as well as the probabilistic features of the tasks, he can vary his tactics so as to maximise the likelihood of a high overall score.

At the very least, surely someone should by now have redrawn the curve of "Probability of Arrival versus Average Speed" for a 300 km task in a Libelle (for example) on a typical Australian summer day, so that pilots attempting Gold Distance could be shown how likely they are to succeed, and how their chances will be affected by their choice of inter- thermal speeds.

Don't statisticians ever take up gliding?

Of course, "The Armchair Pilot" himself, A. W. F. Edwards, is a statistician who took up gliding. In the wider world he is known as "Fisher's Edwards".

Editorial note:

The following expression:
$s=Av^3+B/v$
was given incorrectly in *Australian Gliding* as:
$s=Av^2+B/v$
The following expressions: $(1/d)exp(-x/d)$ and $(1-exp(-h/(2d\ (AB)^{0.5})))^n$
were given incorrectly in *Sailplane and Gliding* as:
$1/d.\ exp.\ (-x/d)$ and $(1-exp\text{-}h/2d\ AB)^n$